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DISCUSSION

DISCUSSION OF "MODAL STIFFNESS OF A PRETENSIONED CABLE NET", BY C. R. CALLADINE. *INT. J. SOLIDS STRUCTURES* 18, 829–846 (1982)

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In his paper "Modal Stiffnesses of a Prestressed Cable Net", Mr. C. R. Calladine has presented an interesting method of studying the behaviour of pretensioned suspension cable nets. The method is based on his work on the analysis of tensegric bodies[1]. Mr. Calladine is referring to tensegric structures and suspension cable net roofs as similar structures. It should be stressed that the concept "tensegrity" was coined by Fuller[2] to name the structures developed by Snelson; regular bodies in which the edges are cables pretensioned by bars that are connected at both of their ends to the body nodes and where no bar touches another. Because the inner volume of the tensegric body is full of bars functional use of the inner space is impossible. There were attempts to shift the bars from occupying most of the inner space of the tensegric body to form the so-called tensegric shell in which the free inner space can be of functional use.

First attempts were made by Fuller[3] and Pugh[4] which led to limited types of tensegric shells. The general principle of composing the so-called tensegric nets[5] which lead to an unlimited type of tensegric shell has already been presented by the author. A spherical tensegric shell is shown in Fig. 1.

Fuller defined "tensegrity" as a cable net supported by compression elements. Using this definition also suspension cable net roofs are falling within the category of tensegric shells as the saddle-shape cable net discussed by Mr. Calladine. It is true that from analysis point of view both structure are similar but from an engineering point of view, possible geometrical configurations, transformation of forces, etc. they are totally different. Therefore, there is a trend which should be encouraged to refer to one type of structure as cable net roof and to the second one as tensegric shell.

Mr. Calladine is proposing to study the behaviour of tensegric shells and cable roofs by considering the number of degrees of freedom and the number of independent states of self stress. The number of independent states of self stress is crucial for the analysis. Beside the statement that in the saddle-shaped cable net there is precisely one state of self-stress and calling intuition to understand that in the so-called "loose" assembly the number of independent plates of self stress is zero, there is not thorough discussion in the matter.

Another method of studying the behaviour of tensegric shells and cable net roofs by considering the equilibrium equations was proposed by the author[6, 7]. Where an assembly of m cables and bars is designed to fit into a certain geometrical configuration the uniqueness of this configuration can be studied by using the equilibrium equations of the m modes. These equations take the form of:

$$\tilde{A}\mathbf{P} = \mathbf{Q} \tag{1}$$

in which $\tilde{A}$ is the structure geometrical matrix related to the nodal co-ordinates, $\mathbf{P}$ is the vector of the inner forces in the m members and $\mathbf{Q}$ is the vector of the n external forces acting at the n nodes. It is assumed that the structure reactions can be found by using $\mathbf{P}$ directly. The assembly of cables and bars has a unique geometrical configuration which

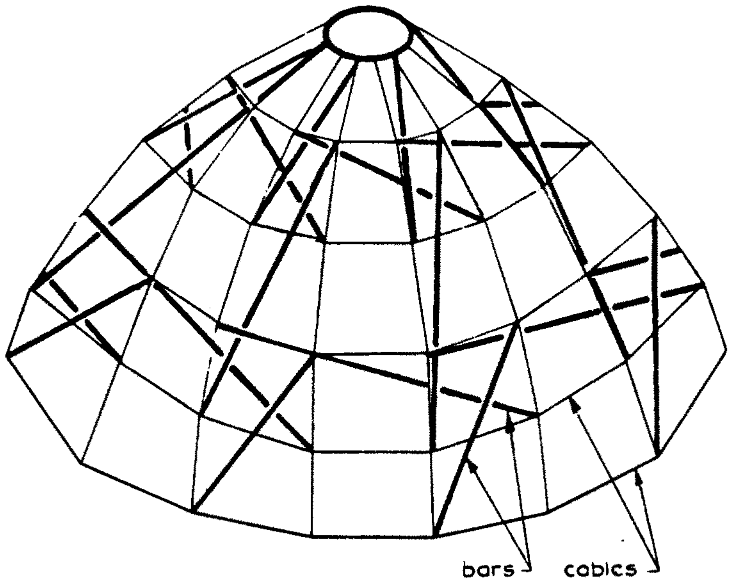

Fig. 1. A tensegric spherical shell.

indicates that the assembly is stable where the following two conditions are satisfied:

(1) It can be prestressed.

(2) Prestressing induces tension in the cables.

The possibility of prestressing is conditioned by the equilibrium of the inner forces acting at the nodes, which implies:

$$\tilde{A}\mathbf{P}_t = 0 \tag{2}$$

in which $\mathbf{P}_t$ is the vector of the prestressed forces in the structure members.

Equation (2) is satisfied where:

$$\text{Ranh } \tilde{A} < m. \tag{3}$$

In case 1, where the number of internal inner forces m is larger than the number of equilibrium equations n, the so-called indeterminate structure, this requirement is always satisfied.

In case 2 where the number of unknown inner forces is equal to the number of equilibrium equations, the so-called determinate structure equation condition[3] implies that:

$$\det \tilde{A} = 0. \tag{4}$$

In case 3 where the number of unknowns is smaller than the number of equations, the so-called statically unstable structure this condition implies that a series of determinents should be equal to zero.

It was shown how an assembly of bars and cables can be designed to fit into a desirable stable geometrical shape. Where condition 3 is satisfied the members have specific lengths. If one member is larger or shorter than required it, in most cases, implies that the assembly cannot be fitted into a unique geometrical configuration and so the new assembly is not stable—it is a mechanism. Mr. Calladine's reference to a "tight" or a "loose" assembly where one member is shorter or larger, is rather misleading.

The forces induced in the structure members by prestressing can be found by using eqn (2). The number of independent states of self-stress is equal to the number of members in which the inner forces is to be assumed in order to predict the other members inner forces. It can be seen that in case 1 this number is equal to m-n and in cases 2 and 3 it is at least 1. Only structures where tension is induced into the cables are feasible. Where the structure is prestressed, the change in the members' length due to their elasticity effect the assembly geometrical configuration. In every case the cables are prestressed to the extent that under expected loads a certain amount of tension is left remaining in them. Also, the case where an external load is applied to the structure was studied. It was found that in the case of an indeterminate structure the cable-net roof or a tensegric shell can sustain external loads in its prestressed geometry. The nodal displacements are due to the members elasticity only, as in ordinary reticulated shells. In the case of determinate and statically unstable structures the structure cannot sustain general external loads in its prestressed geometry. There will be geometrical distortions, not only due to the members elasticity, until equilibrium at the various nodes is achieved. This geometrical distortion is finite and would not lead to a total collapse of the structure. Only under the so-called fitted loads the nodal displacements are due to the members elasticity only. Because in the case of geometrically unstable structures the displacements are finite it seems to be inaccurate to refer to this type of tensegric shells or cable net roofs as mechanisms. Only structures that cannot keep their geometrical configuration and external load may cause to a total collapse of the structure, should be defined as mechanisms. For example, a geometrically unstable assembly of cables and bars that cannot be fully prestressed. The saddle-shape cable net roof discussed by Mr. Calladine is a geometrically unstable structure and not a mechanism. It is important to realize that by designing this cable net roof, with more cables, may be by adding diagonals, the net is an indeterminate structure and the prestressed net is acting as an ordinary reticulated shell.

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